# Fermionic Superfluidity in 1D Cold Atoms

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# Fermionic Cold Atoms

- Fermionic cold atoms with spin $S = N - 1/2$. Ex: $^{40}$K, with $S = 9/2$ or $^{132}$Cs, $^{9}$Be, $^{137}$Ba with $S = 3/2$.
- $2S + 1 = 2N$ states $c^\dagger_\alpha$, $\alpha = -S, ..., +S$
- Model Hamiltonian $\rightarrow$ all possible even spin $J$ pairings:

$$P^\dagger_{Jm} = C^{(\alpha,\beta)}_{(JM)} c^\dagger_\alpha c^\dagger_\beta$$

$$\boxed{\mathcal{H} = -t \sum_{i,\alpha} (c^\dagger_{i+1,\alpha} c_{i,\alpha} + HC) + \sum_{J=0,..,2N-2} U_J \sum_{i,M} P^\dagger_{i,JM} P_{i,JM}}$$

$$\boxed{SU(2) \rightarrow SU(2N)}$$

# Spin 1/2 and the Hubbard Model

- J=0 singlet pairing: $\boxed{P^\dagger_{i,00} = c^\dagger_{i,1/2} c^\dagger_{i,-1/2}}$
- $U_0 < 0$, coexisting SUPRA and $2k_f$ CDW instabilities:

- $\mathcal{O}_{CDW}(x) = e^{2ik_F j} \sum_\alpha c^\dagger_{j,\alpha} c_{j,\alpha}$

- $\mathcal{O}_{SUP}(x) = c^\dagger_{i,1/2} c^\dagger_{i,-1/2}$

$$< \mathcal{O}^\dagger_{CDW}(x) \mathcal{O}_{CDW}(0) > \sim {\rho_n}^2 / x^{\alpha_c}$$

$$< \mathrm{O}^\dagger_{SUP}(x) \mathcal{O}_{SUP}(0) > \sim {\rho_s}^2 / x^{\beta_c}$$

$$\boxed{\rho_n = \rho_s \sim < \cos(\sqrt{2\pi}\Phi_s) >}$$

# Spin 3/2

- 4 states $c^\dagger_\alpha$, $\alpha = \pm 3/2, \pm 1/2$
- Both J=0 and J=2 pairings:

- $P^\dagger_{00} = (c^\dagger_{3/2} c^\dagger_{-3/2} - c^\dagger_{1/2} c^\dagger_{-1/2})/\sqrt{2}$

- $P^\dagger_{2M=(2,1,0,-1,-2)} =$
$(c^\dagger_{3/2} c^\dagger_{1/2}, c^\dagger_{3/2} c^\dagger_{-1/2}, (c^\dagger_{3/2} c^\dagger_{-3/2} + c^\dagger_{1/2} c^\dagger_{-1/2})/\sqrt{2}, c^\dagger_{1/2} c^\dagger_{-3/2}, c^\dagger_{1/2} c^\dagger_{3/2})$

$$\boxed{\mathcal{H} = -t \sum (c^\dagger_{i+1,\alpha} c_{i,\alpha} + HC) + U_0 \sum P^\dagger_{i,00} P_{i,00} + U_2 \sum P^\dagger_{i,2m} P_{i,2m}}$$

$$\boxed{SU(2) \rightarrow SU(4)}$$

# Symmetry Analysis

- U(1) Charge (density) with $N = \sum_{i,\alpha} c^{\dagger}_{i,\alpha} c_{i,\alpha}$

Wu and Zhang 2004 $\boxed{SO(5)}$

- $SU(4) = \mathcal{L} \oplus \mathcal{N}$ with $[\mathcal{L}, \mathcal{L}] \in \mathcal{L}, [\mathcal{L}, \mathcal{N}] \in \mathcal{N}$ and $[\mathcal{N}, \mathcal{N}] \in \mathcal{L}$.
- $N^a_i = \sum c^{\dagger}_{i,\alpha} \Gamma^a_{\alpha,\beta} c_{i,\beta}$; $a = (1, ..., 5)$
- $L^{ab}_i = -i/2[N^a_i, N^b_i] \in \mathrm{SO}(5)$

$$\boxed{\mathcal{H} = -t \sum_{i,\alpha} (c^{\dagger}_{i+1,\alpha} c_{i,\alpha} + HC) + U/2 \sum_i (N_i)^2 + V \sum_i \sum_a (N^a_i)^2 + \mathrm{Cte}}$$

$(\mathrm{U} = \frac{3U_0 + 5U_2}{4}$ , $\mathrm{V} = \frac{U_0 - U_2}{2})$

- SO(5): $[\sum_i L^{ab}_i, \mathcal{H}] = 0$

# Conserved Charges

- SU(4) embeding $\rightarrow$ 3 Cartans generators:$(\mathcal{S}^z, \sigma, \tau)$
  - Spin $\mathcal{S}^z = \sum_i (\frac{3}{2} n_{i,3/2} - \frac{3}{2} n_{i,-3/2} + \frac{1}{2} n_{i,1/2} - \frac{1}{2} n_{i,-1/2})$
  - Pseudo Spin $\sigma = \frac{1}{2} \sum_i (n_{i,3/2} - n_{i,-3/2} + n_{i,1/2} - n_{i,-1/2})$
  - Pseudo Orbital $\tau = \frac{1}{2} \sum_i (n_{i,3/2} + n_{i,-3/2} - n_{i,1/2} - n_{i,-1/2})$

Where $n_{i,\alpha} = c^\dagger_{i,\alpha} c_{i,\alpha}$

- $\mathcal{S}^z$ and $\sigma$ are 2 SO(5) Cartans.
- Pseudo Orbital $\tau$ is not conserved.

# Weak Coupling and Continuum Limit

Generic fillings $\rightarrow$ Spin-charge separation

- Charge-Density: Luttinger Liquid
- Spin sector: $SO(5) \otimes Z_2$ Gross-Neveu Model:

$$\boxed{\mathcal{H}_s = \mathcal{H}_{SU(4)_1} + g_{||} L_R^{ab} L_L^{ab} + g_\perp N_R^a N_L^a}$$

$$g_{||} = U_0 + U_2,\ g_\perp = 3U_2 - U_0$$

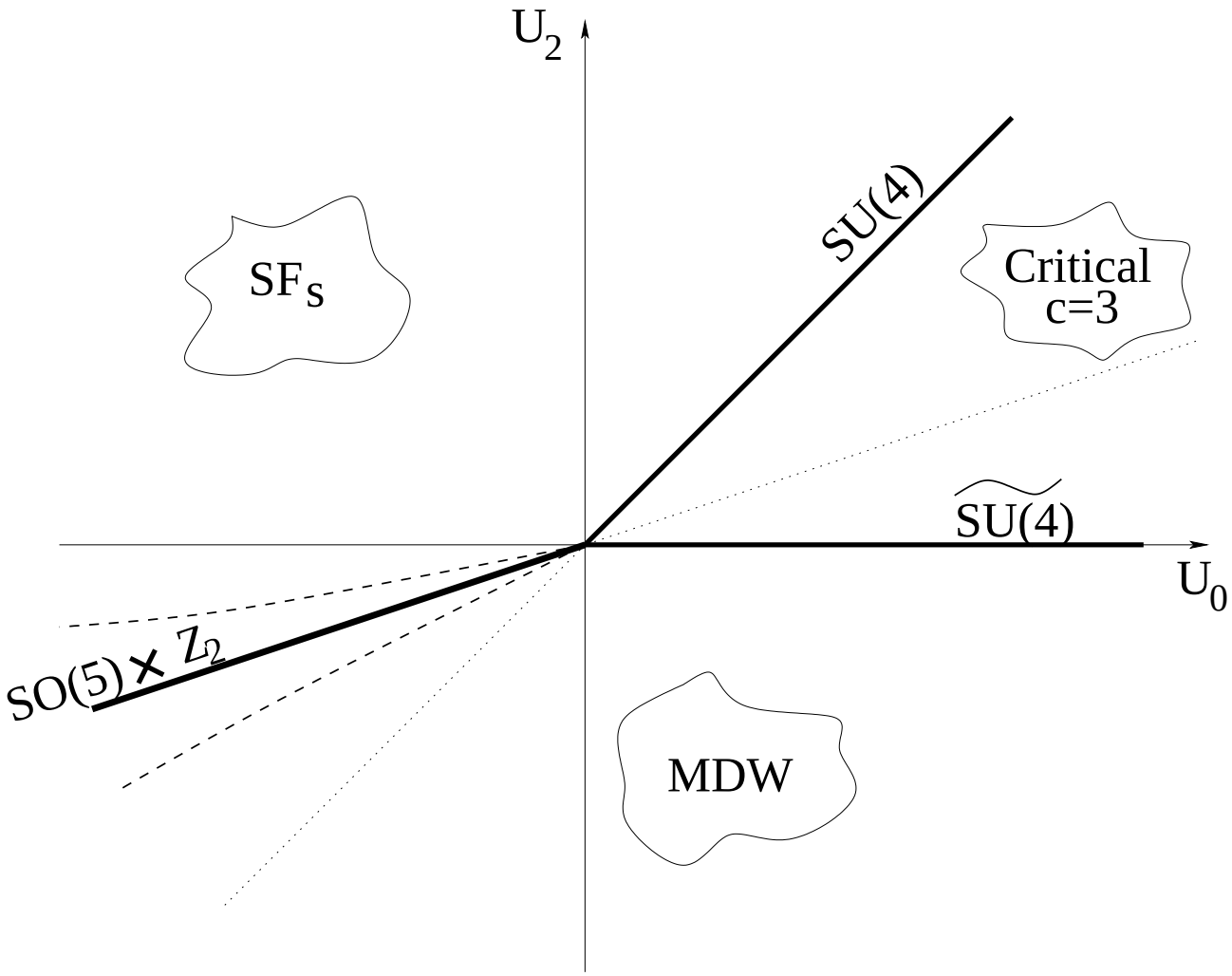
- $g_{||} = g_\perp \Leftrightarrow U_2 = U_0$, SU(4) symmetric
- $SU(4) = L \oplus N$ with $[L, L] \in L, [L, N] \in N$ and $[N, N] \in L$.
- $[L, \mathcal{H}_s] = 0 \Rightarrow \boxed{SO(5)}$
- $N_{R(L)}^a \Rightarrow -N_{R(L)}^a \Rightarrow \boxed{\mathrm{Z}_2}$

# Renormalization Group

Neglect velocity anisotropy $\longrightarrow$ Gerganov et al 2000

$$
\begin{aligned}
\dot{g}_{||} &= 4\left(g_{||}-2\right)^2\left[3\left(\frac{g_{||}}{g_{||}^2-4}\right)^2+\left(\frac{g_{\perp}}{g_{\perp}^2-4}\right)^2\right] \\
\dot{g}_{\perp} &= 16\,\frac{g_{\perp}\left(g_{\perp}^2-2g_{||}\right)}{\left(g_{\perp}^2-4\right)\left(g_{||}+2\right)} \\
g &\longrightarrow g/2\pi v
\end{aligned}
\tag{2}
$$

- $|g| \leq 2 \rightarrow$ Weak coupling Regime
- Dynamical Symmetry Enlargement (DSE) of SU(4) : $g_{||}^* = g_{\perp}^*$ and $g_{||}^* = -g_{\perp}^*$


U2
SU(4)
SFs
Critical
c=3
SU(4)
U0
SO(5) x Z2
MDW

# Normal SU(4) CDW Phase

SU(4) Gross-Neveu Model

$$\boxed{\mathcal{H}_s^* = \mathcal{H}_{SU(4)_1} + g^*(L_R^{ab} L_L^{ab} + N_R^a N_L^a)}$$

- $2k_F$ CDW instability: $\boxed{\rho_n \neq 0 \ \rho_s = 0}$
- Pseudo-Orbital $\tau = \int(\tau_R + \tau_L)$ becomes a good quantum number
- Excitations : m: $\square$ , $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ and $\sqrt{2}$ m : $\begin{smallmatrix}\square\\\square\end{smallmatrix}$

# Dual Superfluid SU(4) Phase

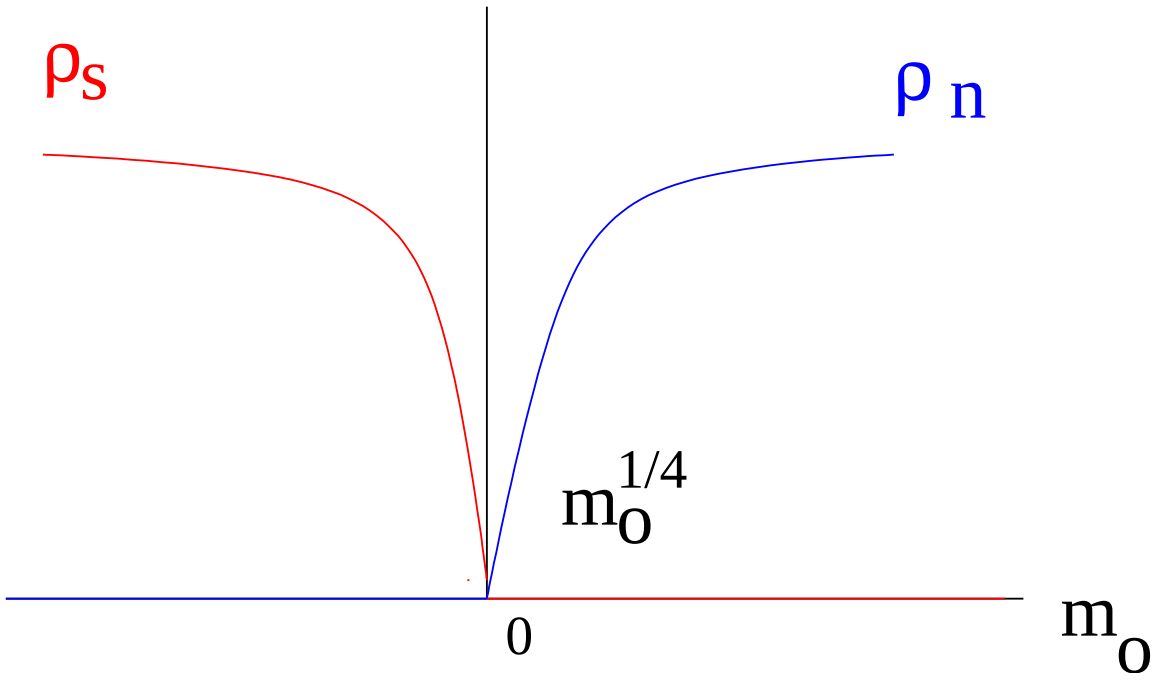
Dual SU(4) Gross-Neveu Model

$$\mathcal{H}_s^* = \mathcal{H}_{SU(4)_1} + g^*(L_R^{ab} L_L^{ab} - N_R^a N_L^a)$$

- $Z_2$ Duality : $N_R^a \to N_R^a, N_L^a \to -N_L^a, g_\perp \to -g_\perp$
- $\rho_s \leftrightarrow \rho_n$, Singlet Superfluid instability: $\rho_s \neq 0 \; \rho_n = 0$
- Pseudo-Orbital current $\tau = \int(\tau_R - \tau_L) \Rightarrow$ Chiral Superfluid

## Quantum Ising transition and spectral cross-over

- Quantum Ising transition on the $Z_2$ self-dual line $g_\perp = 0 \Leftrightarrow U_0 = 3U_2$
- In the quantum critical region the SU(4) spectrum split:
  - m $\square$ $\Rightarrow \mathcal{S} = 3/2 \rightarrow m_{3/2}$
  - $\sqrt{2}$ m $\begin{array}{c}\square\\\square\end{array}$ $\Rightarrow (\mathcal{S} = 0) + (\mathcal{S} = 2) \rightarrow m_0 < m_2$
- $m_0 \rightarrow 0$ singlet bound states condense


ρs
ρ n
m_o^{1/4}
0
m_o

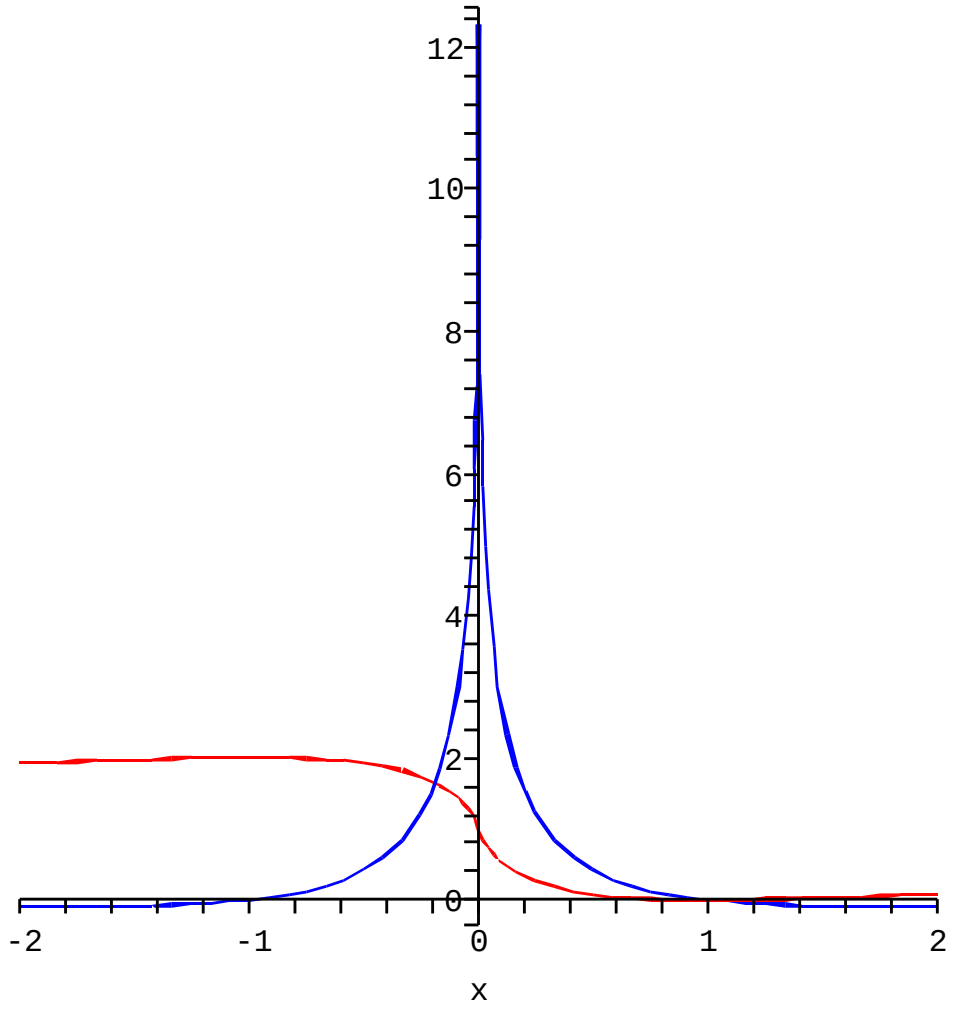
# Response Functions and Phase Coherence

$$\mathcal{H}_{\text{ext}} = \sum_{i,a} \mu_a N_i^a$$

- $\mu\tau = \mu\frac{1}{2}\sum_i (n_{i,3/2} + n_{i,-3/2} - n_{i,1/2} - n_{i,-1/2})$ , $< \tau >= \partial E(\mu)/\partial\mu$
- Normal CDW Phase $< \tau >\sim 0$
- Superfluid Chiral Phase $< \tau >\neq 0$

- $\int(\tau_R - \tau_L)$ is conserved: $[\tau, \Phi] = i$,$\Phi$ is locked

- In the quantum critical region $\Xi(m_0) = \partial^2 E(\mu)/\partial\mu^2 \sim \log|m_0|$


12
10
8
6
4
2
0
-2
-1
0
1
2
x

# Concluding Remarks

- Spin 3/2 Superfluid and Normal CDW phases are dual. They are separated by a quantum phase transition of the Ising type.
- Superfluidity is of chiral nature.
- Spins $S = N - 1/2$ ; N $> 2 \Rightarrow$ SU(2N) embeding. Superfluid and CDW phases are dual with quantum phases transtions in between.
- Need a Numerical check Azaria, Boulat, Caffarel and Lecheminant 2004
- These properties could be possibly observed in experiments.