Theory of anomalous heat transport in 1D spin systems

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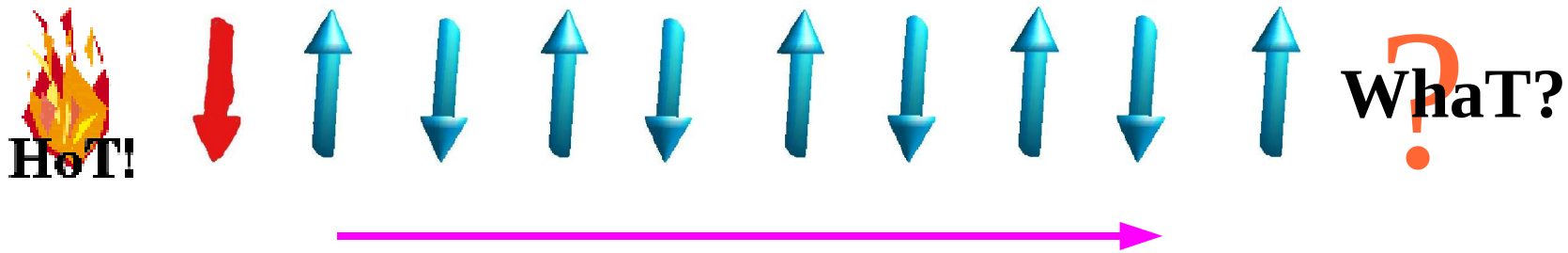
Thermal transport in quasi-one-dimensional quantum magnets:

- Excitations: spin bosons and phonons
- Scattering mechanisms: spin-phonon, spin-impurity

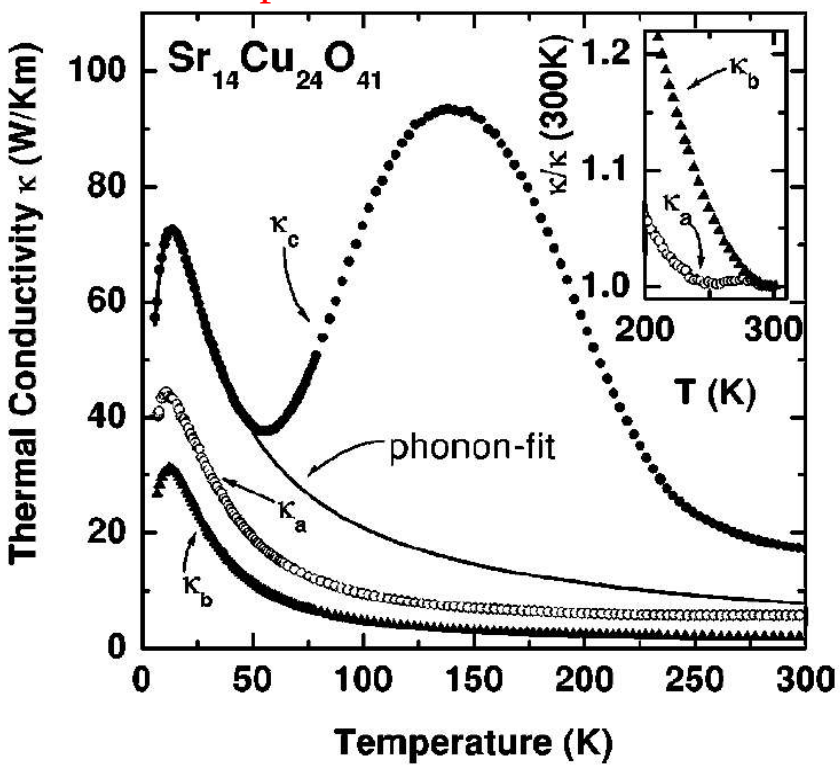


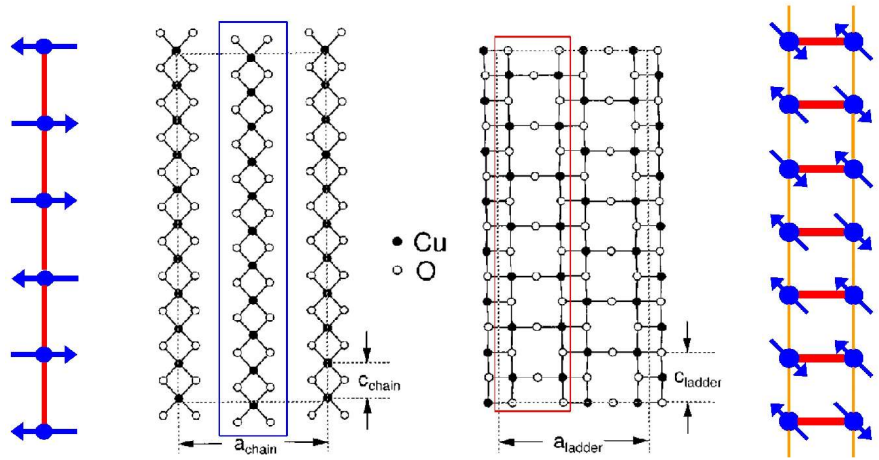
cond-mat/0407256, cond-mat/0407257

1D heat transport, experiments

C. Hess *et.al.*, Phys. Rev. B **64**, 184305 (2001)

"c" = spin-chain direction, $J \sim 2000K$





Cuprates:

1D family Sr_2CuO_3, $SrCuO_2$, $Sr_{14}Cu_{24}O_{41}$

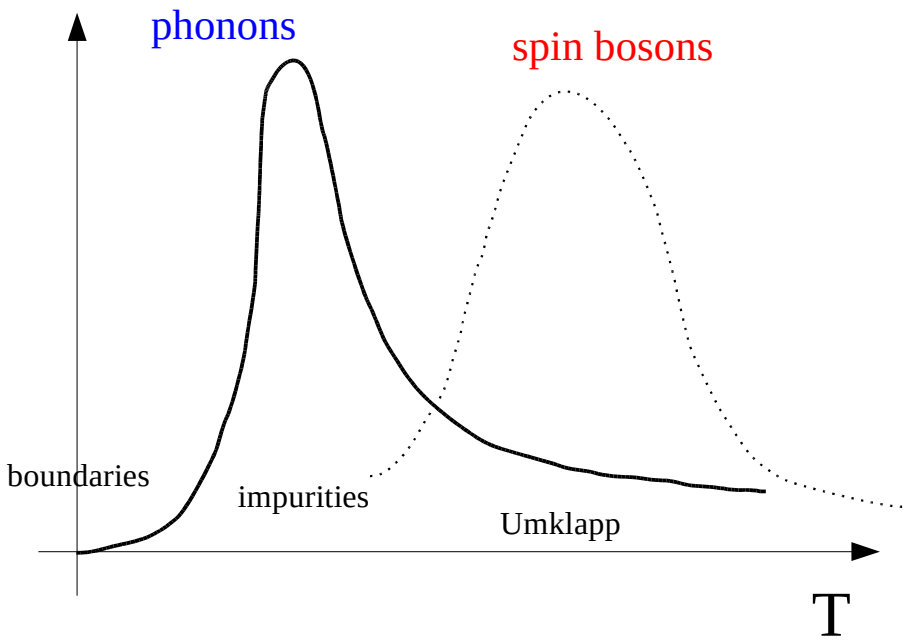
spin-chains, spin-ladders

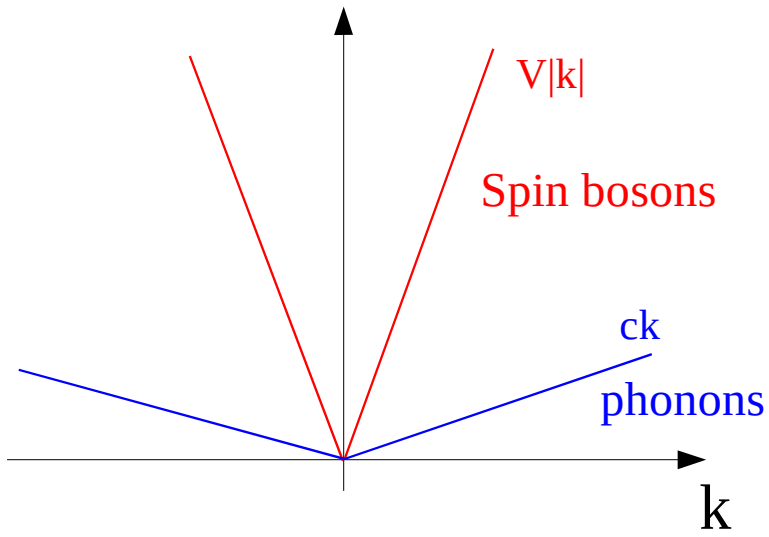
Comparable to of metallic systems!

Spin-phonon interaction I

There are two small parametrs in the problem:

- Spin-lattice coupling is very weak => *use of Boltzmann equation is justified*
- c/v << 1 => *T is still "low" for spin bosons when it is "high" for phonons, phonons can be treated as being in local equilibrium*



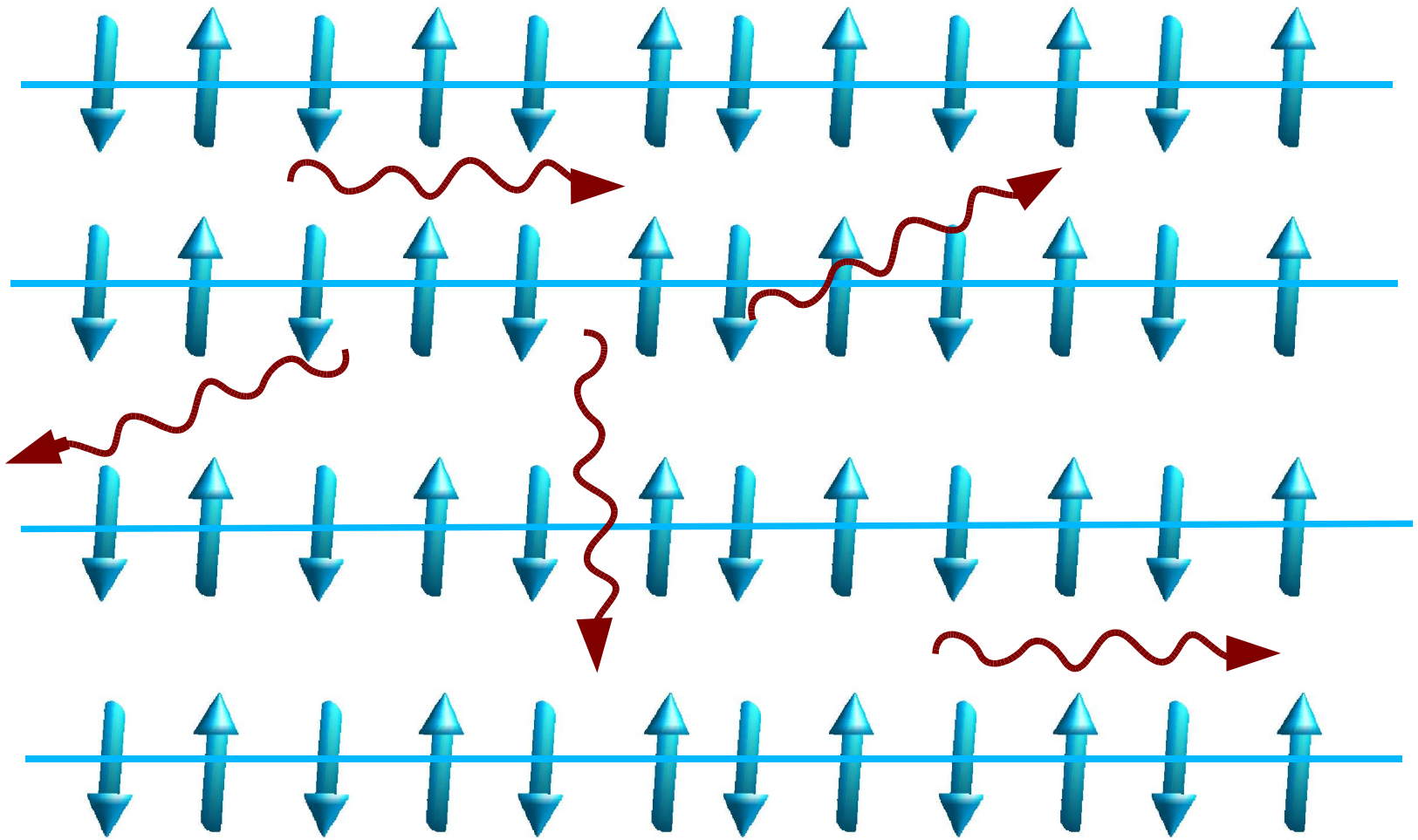


phonons are close to local equilibrium:

$$n_k = n_k^0$$

small parameter c/v << 1:
fast spin bosons
slow phonons

Model picture



Spin chains + 3D isotropic phonons

Spin chain

1D Heisenberg model: $$H_{\mathrm{XXX}} = J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

Jordan-Wigner fermions (domain walls):

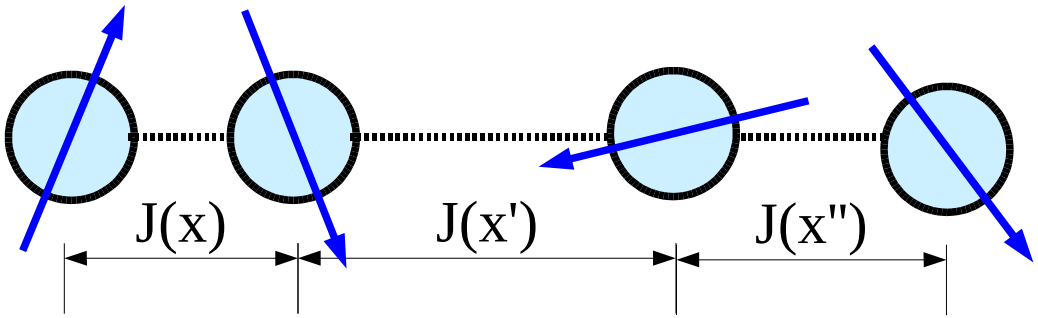
$$H_{JW} = -\frac{J}{2} \sum_i \left(\psi_i^\dagger \psi_{i+1} + \mathrm{h.c.} + \frac{1}{2} \left(2\psi_i^\dagger \psi_i - 1 \right) \left(2\psi_{i+1}^\dagger \psi_{i+1} - 1 \right) \right)$$

Bosonization (domain wall density):

$$H_0 = \frac{v}{2} \int dx \left(\mathcal{K} \left(\partial_x \Theta \right)^2 + \mathcal{K}^{-1} \left(\partial_x \Phi \right)^2 \right)$$

$$H_0 = v \sum_k |k| b_k^\dagger b_k$$ gas of free spin bosons

Spin-phonon interaction



Phonons induce local variation of J and v
(v is proportional to J)

$$H_{\text{int}} = -\frac{\delta v}{\sqrt{N}} \sum_{kk'\mathbf{P}\ell} \frac{P_{\|} kk'}{\sqrt{8 m_i \omega_{\mathbf{P}\ell} kk'}} \left(\hat{\xi}_{\mathbf{P}\ell}\right)_x \left(a^{\dagger}_{\mathbf{P}\ell} b^{\dagger}_{k} b_{k'} \delta_{P_{\|},k'-k} + \text{h.c}\right)$$

Momentum relaxation

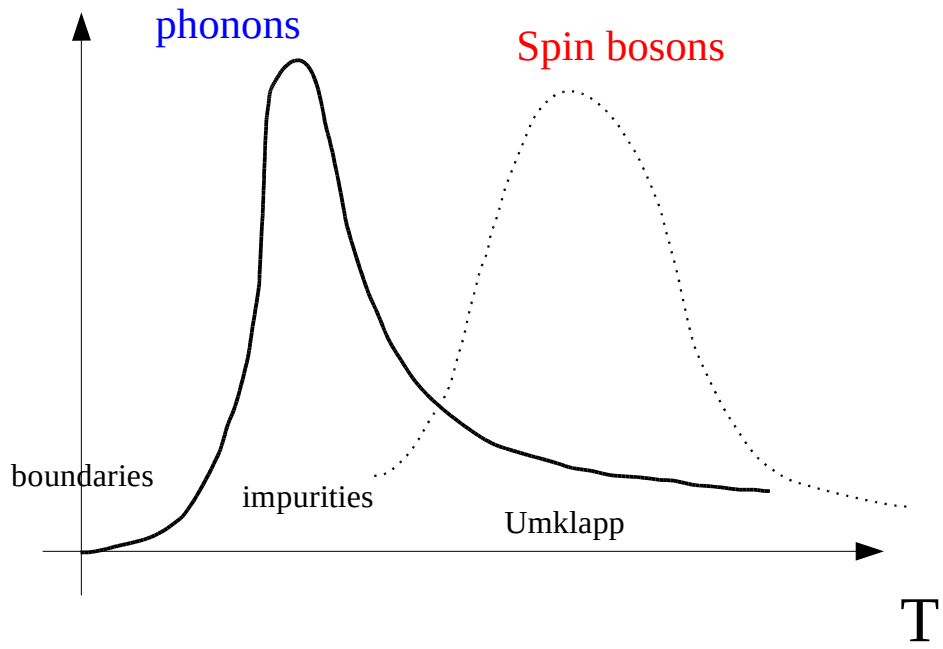
Umklapp scattering: $k=k'+p+G$

exponentially suppressed at low temperatures.

"Low" temperatures: for phonons $T << {}_D$

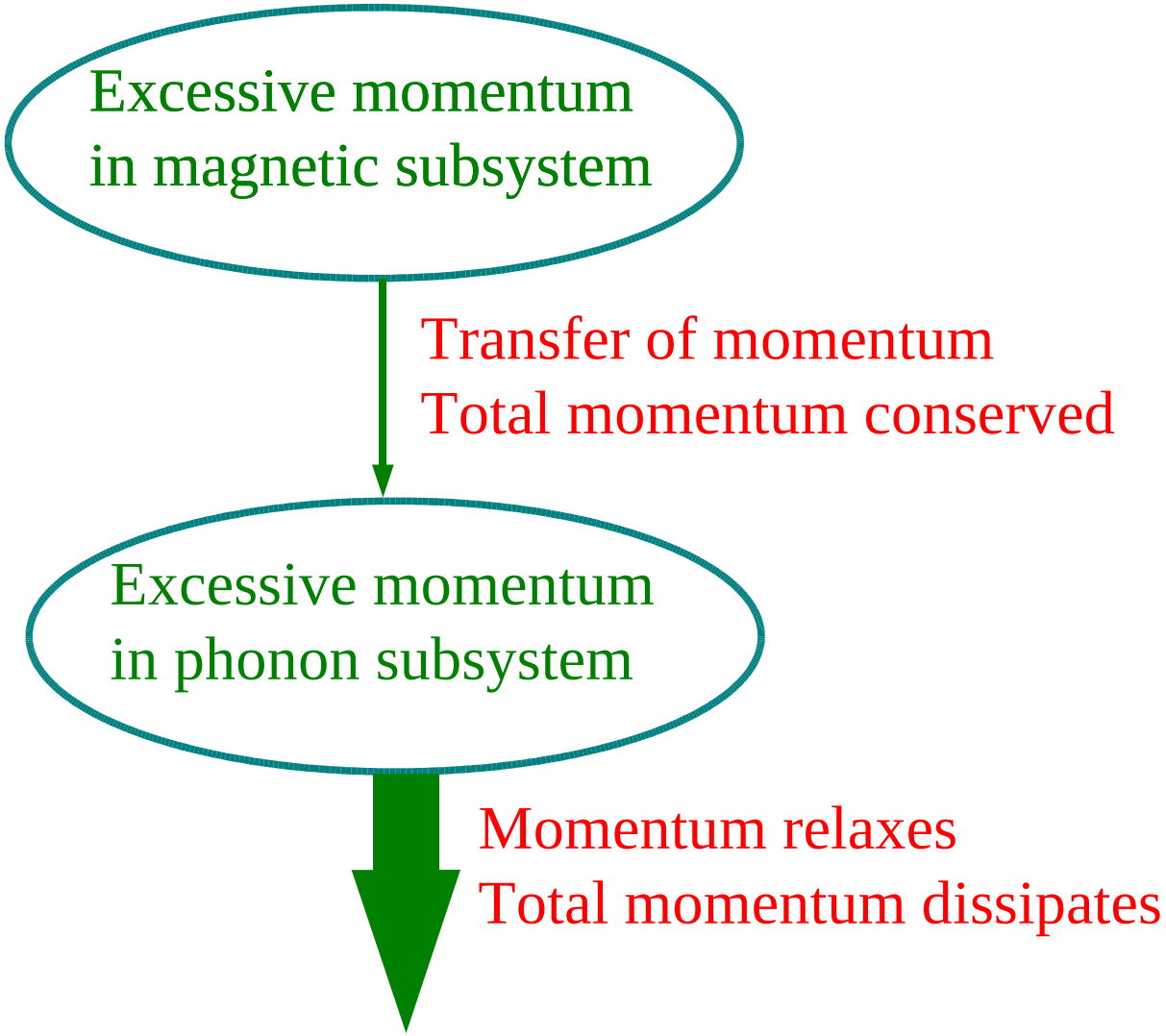
for spin bosons $T << J$

Impurity scattering



Momentum relaxes though phonon-phonon scattering!

Bottle-neck


Excessive momentum
in magnetic subsystem
Transfer of momentum
Total momentum conserved
Excessive momentum
in phonon subsystem
Momentum relaxes
Total momentum dissipates

Boltzmann transport theory

Heat conductivity: $\kappa_s \sim \sum_k v^2 \tau_k c_k$

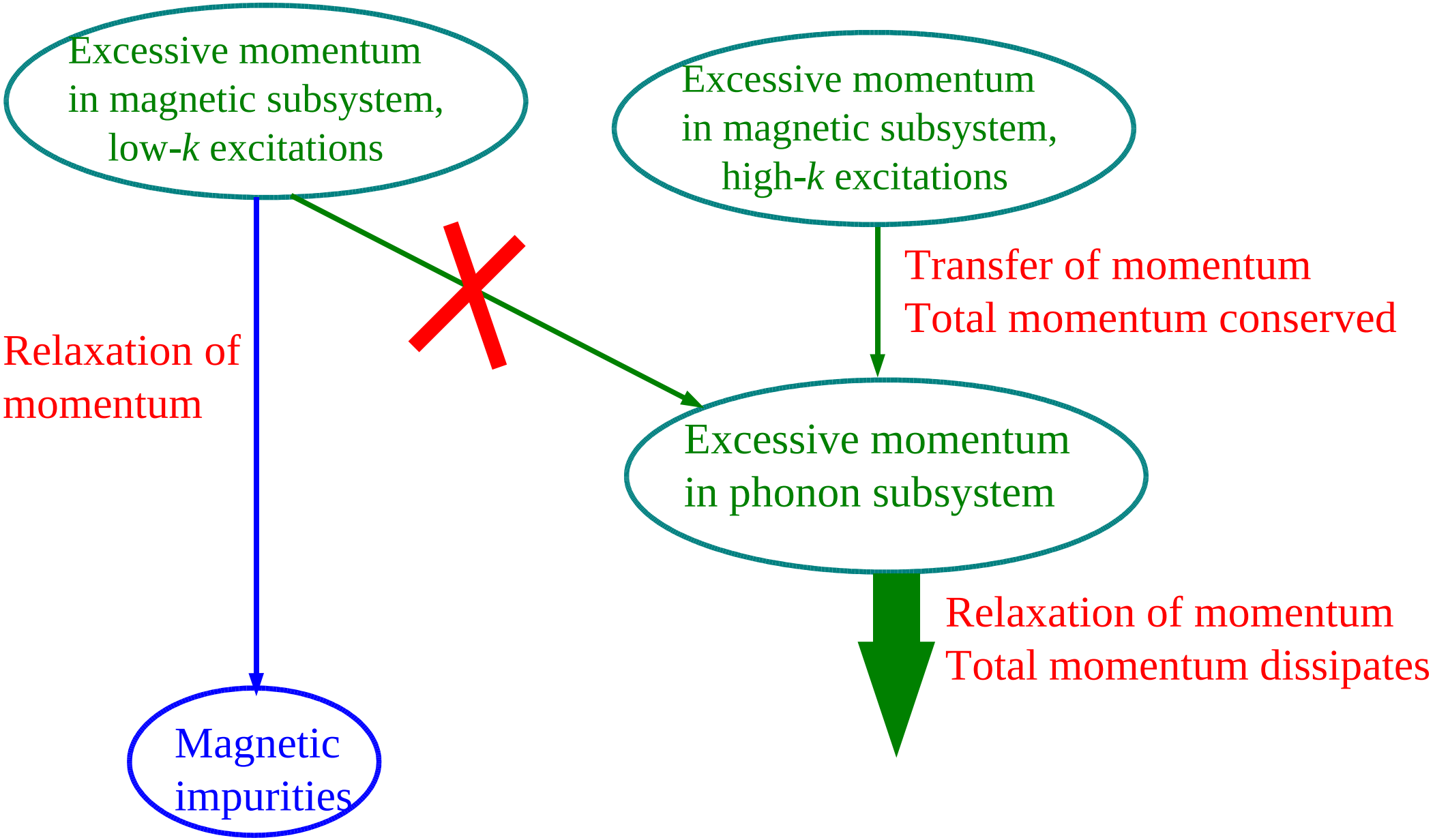
Relaxation time τ: $\frac{d}{dt}(f_k - f_k^0) = -\frac{f_k - f_k^0}{\tau_k}$

$$\tau_k^{-1} \sim A \frac{T^3 k^2}{v^3}$$

$$\boxed{\kappa \sim \frac{v^5}{A T^3} \int_0^{T/v} \frac{dk}{k^2}}$$

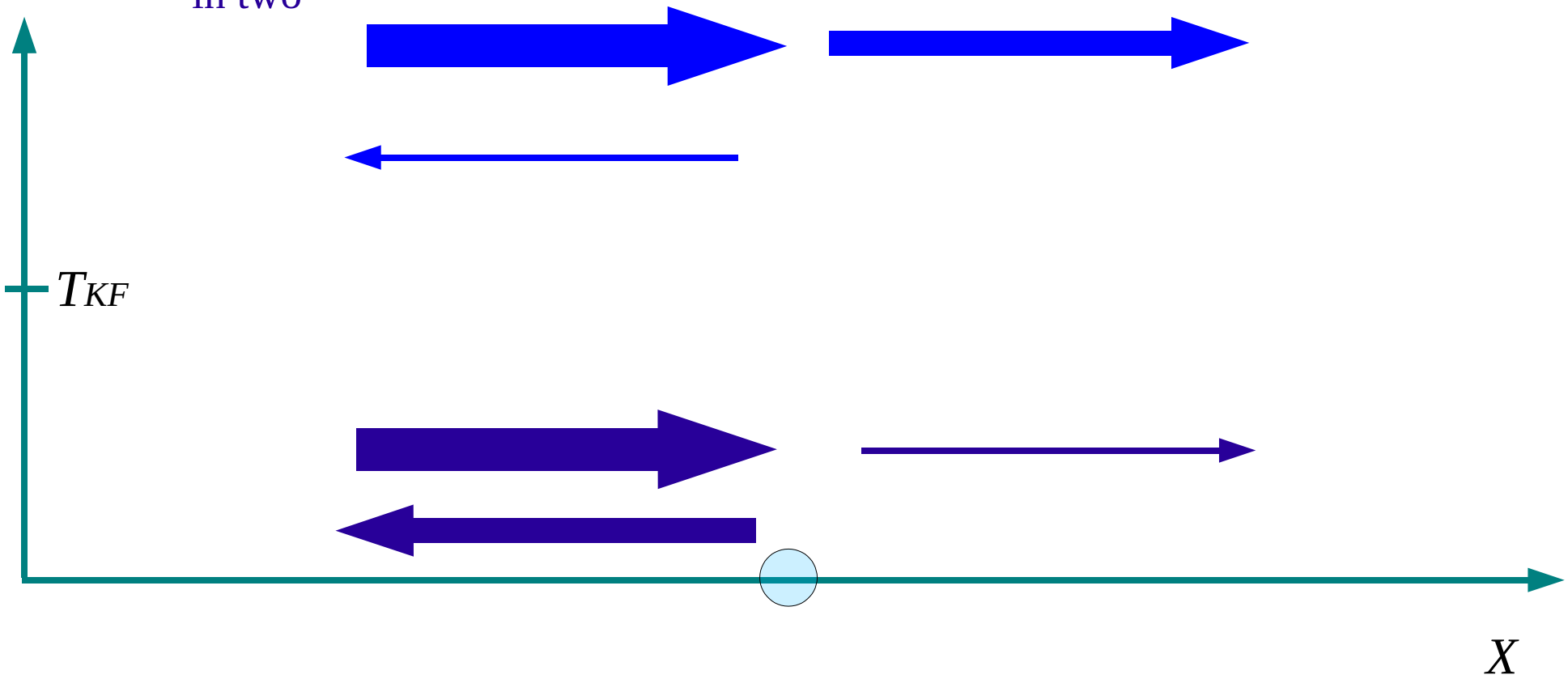
Diverges!

Bottle-neck II



Impurity scattering

- Impurities for 1D spin bosons are very effective scatterers (unlike Rayleigh scattering in 3D) $\cos(\sqrt{2\pi}\Phi(x_0)+k_F x_0)\, vs. (\nabla u(x_0))^2$
- They lead to the new scale (Kane-Fisher temperature) $T_{KF} \sim J\left(\frac{V_{imp}}{J}\right)^{\frac{1}{1-K}} = \frac{V_{imp}^2}{J}$ below which the impurity acts as an ideal backward scatterer, effectively, cutting the chain in two



Impurity scattering II

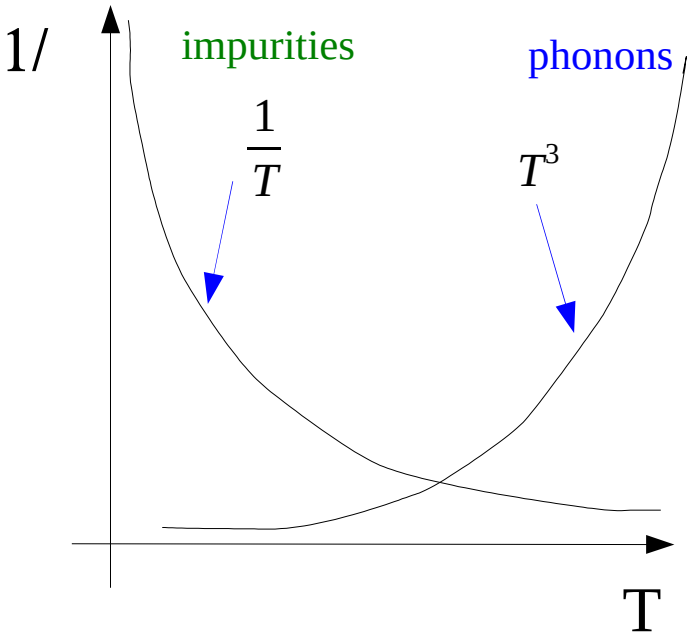
- At $T >> T_{KF}$ the impurity scattering can be treated perturbatively
- Transport relaxation time = spin boson life-time
- Calculate spin boson Green's function and extract the life-time

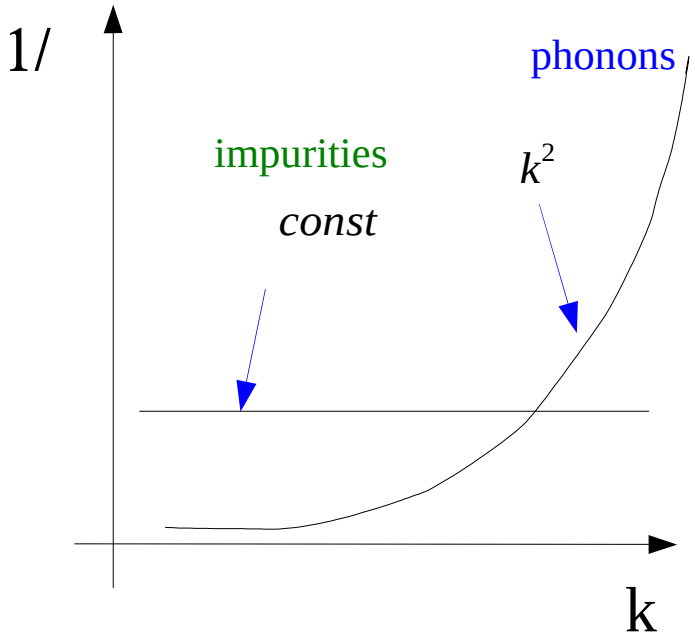
$$\boxed{\tau_{imp}^{-1} \sim \frac{n_{imp}\, a\, V_{imp}^2}{T}}$$

independent of k

Altogether:

$$\tau_k^{-1} \sim \tau_{imp}^{-1} + \tau_{s-ph,k}^{-1}$$



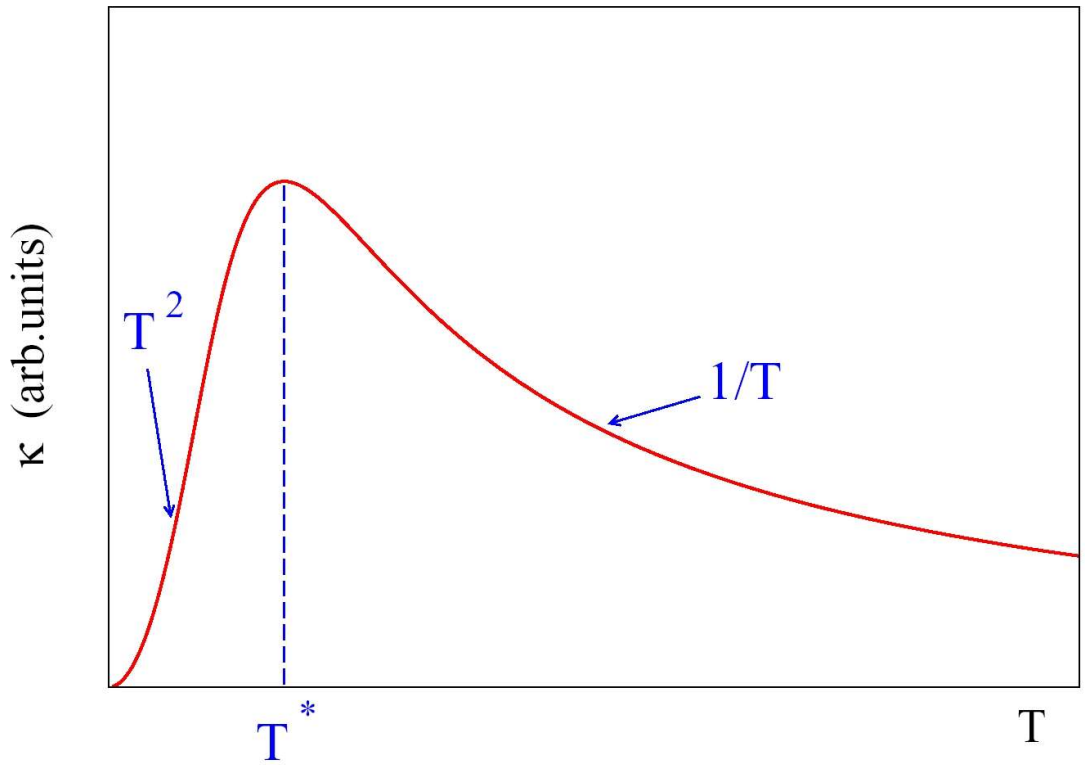


Results

Thermal conductivity

$$\kappa \sim \kappa_{max} S\left(\frac{T}{T_m}\right)$$

$$S(r) = \frac{1}{r}\arctan(r^3)$$



$$T_m \sim \left(n \frac{V_{imp}^2}{A} J\right)^{1/6} \sim n^{1/6}$$

$$\kappa_{max} \sim \frac{v^6}{A T_m^4} \sim n^{-2/3}$$

n - impurity concentration

Bottle-neck III

High and low T

Excessive momentum
in magnetic subsystem,
low-k excitations

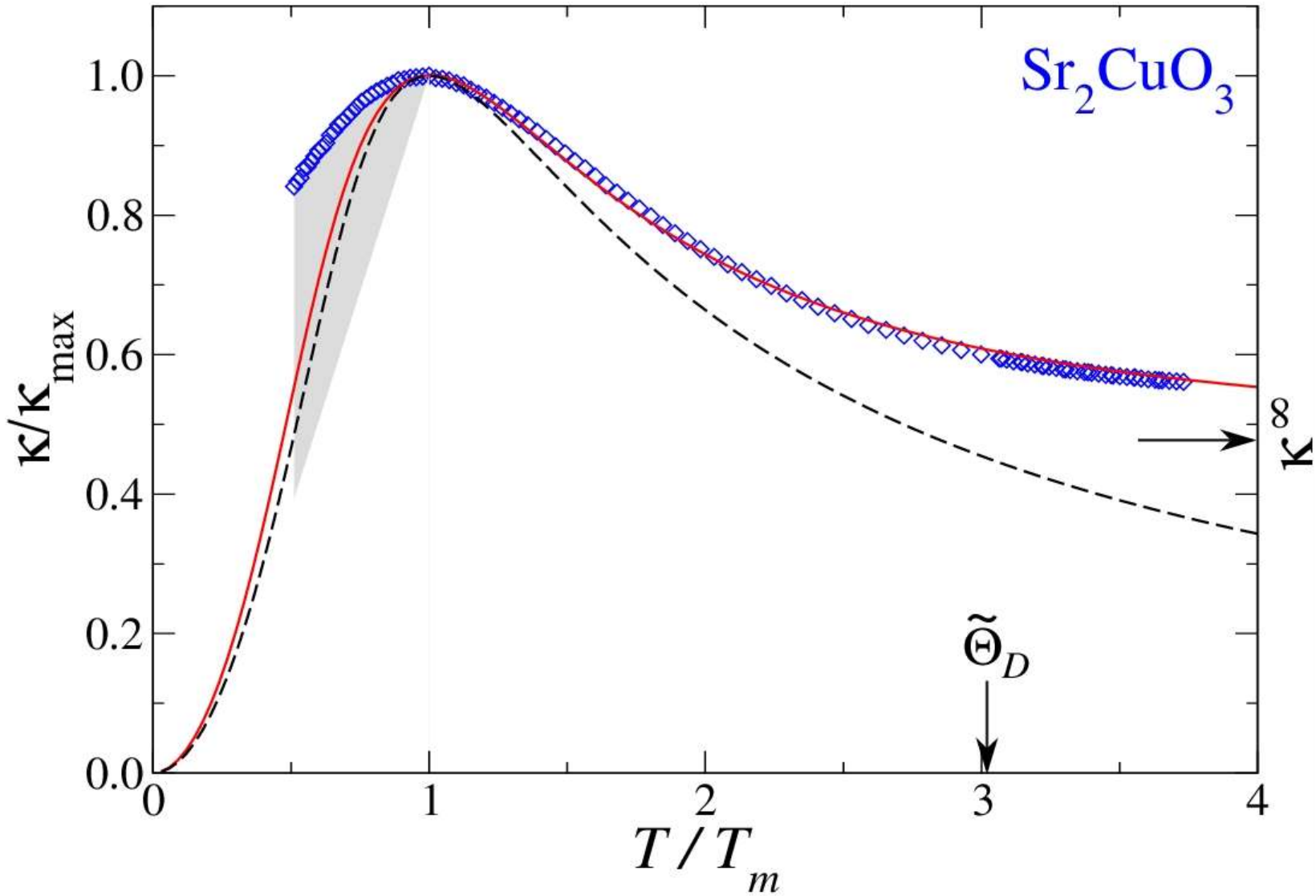
Magnetic
impurities

$T > T_m$ only

Excessive momentum
in magnetic subsystem,
high-k excitations

Excessive momentum
in phonon subsystem

Results II



* Data are courtesy of Dr. A. Sologubenko

Conclusions

- "normal" scattering process of spin bosons on phonons is the dominant relaxation mechanism for the high energy spin bosons
- impurities are very important at low energies to cut-off the slow relaxing modes
- Spin-phonon and impurity scattering are enough to reproduce the general structure of the magnetic heat conductivity as a function of temperature
- Predictions:

$$\kappa_s(T) \propto \begin{cases} T^2 & \text{for } T \ll T_m \ , \\ T^{-1} & \text{for } T_m \ll T \ll \widetilde{\Theta}_D \\ T^0 & \text{for } T \gg \widetilde{\Theta}_D \ . \end{cases}$$

$$\widetilde{\Theta}_D \approx \Theta_D/4$$

Impurity concentration dependencies:

$$T_m \propto n^{1/6}$$

$$\kappa_s^{max} = \kappa_s(T_m) \propto n^{-2/3}$$

$$\kappa_s^{\infty} \propto n^{-1/2}$$